

Three-Stage Least Squares (3SLS)

Three-Stage Least Squares is a **systems estimation method** for simultaneous equation models. Unlike single-equation methods like 2SLS, it estimates **all equations simultaneously**, improving efficiency when error terms across equations are correlated.

2SLS vs. 3SLS

	2SLS	3SLS
Estimation	Equation-by-equation	All equations
Information Used	Within-equation information only	Full system information
Cross-equation Error Correlations	Ignored	Considered
Efficiency	Lower	Higher

If error terms across equations are **uncorrelated**, 3SLS reduces to 2SLS. In practice, economic relationships often create correlated errors due to omitted variables, making 3SLS more efficient.

Estimation

Prerequisites

- No autocorrelation** within each equation's error term.
- Contemporaneous correlation** between error terms of different equations.
- The system must be **overidentified** (verified via order and rank conditions).

Model Setup

$$y_{im} = z'_{im}\delta + \varepsilon_{im}$$
$$y = (Y\gamma + X\beta) + \varepsilon = Z\delta + \varepsilon$$

Stage 1 (Same as 2SLS)

reduced form:

$$Z = X\pi + v$$

Stage 2 (Same as 2SLS)

structural model:

$$y = \hat{Z}\delta + u$$

Stage 3

1. Estimate the **cross-equation covariance matrix** Σ using residuals from Stage 2:

$$\hat{\sigma}_{ij} = \frac{\hat{u}_i' \hat{u}_j}{n}$$

$$\hat{\Sigma} = \begin{bmatrix} \hat{\sigma}_1^2 & \hat{\sigma}_{12} & \cdots & \hat{\sigma}_{1m} \\ \hat{\sigma}_{21} & \hat{\sigma}_2^2 & \cdots & \hat{\sigma}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{\sigma}_{m1} & \hat{\sigma}_{m2} & \cdots & \hat{\sigma}_m^2 \end{bmatrix}$$

2. Apply **Generalized Least Squares (GLS)** to the entire system. The **3SLS estimator** is:

$$\hat{\delta}_{3SLS} = \left[\hat{Z}' \left(\hat{\Sigma}^{-1} \otimes I \right) \hat{Z} \right]^{-1} \hat{Z}' \left(\hat{\Sigma}^{-1} \otimes I \right) y$$

Reference:

- [3SLS: Three-Stage Least Squares - SPUR ECONOMICS](#)
- 陈强编著.高级计量经济学及Stata应用.第2版.高等教育出版社.2014